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THRESHOLD BASED STOCHASTIC RESONANCE FOR THE BINARY-INPUT TERNARY-OUTPUT DISCRETE MEMORYLESS CHANNELS

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ABSTRACT

In this paper, we discuss the Stochastic Resonance (SR) effect for the binary-input ternary-output (2,3) Discrete Memoryless Channels (DMC). The (2,3) DMC is of great interest for various digital communications channels, including acoustic underwater communication. We focus on the capacity maximization, brought about by additive Gaussian noise, for the (2,3) DMCs. We analyze the capacity of the threshold binary communication channels in the presence of Additive Gaussian Noise (AGN). In the particular case of binary asymmetric channels with symmetric erasures (BAC/SE) we derive the optimum noise level in order to obtain the maximum capacity. The stochastic resonance for the general binary erasure channel is compared with those of binary symmetric channels. For the general case (2, n) DMC, the Pinsker and Helgert capacity bounds were used for the study of the stochastic resonance.

KEY WORDS

Mutual information, stochastic resonance, binary channels with erasures, threshold communication systems

1. Introduction

In this paper, we discuss the Stochastic Resonance (SR) effect for the binary-input ternary-output (2,3) Discrete Memoryless Channels. The (2,3) DMC is of great interest for various digital communications channels, including acoustic underwater communication [12]. We study the SR impact upon the performance limits of distributed underwater wireless sensor networks operating with limited transmitted power and computational environment. In general, the SR phenomenon is a non-linear effect wherein a communication system can transmit the information with improved efficiency in the presence of the additive Gaussian noise (AGN) [1] and [11]. We focus on threshold communication systems due to the underwater environment, noncoherent communication techniques are affected both by noise and threshold level [5].

Various performance metrics in the presence of SR such as signal to noise ratio, mutual information, and channel capacity improvement under a certain range of power noise levels, were discussed in [2]. In [9], [13] and [14], the SR effect is observed on the binary-input binary-output (2,2) DMC capacity, where maximum capacity occurs at an optimal power (variance) value of the AGN in relation to a given threshold level.

Following Chapeau-Blondeau [2], we analyze the physical communication model of the (2,3) DMC for threshold based SR due to additive Gaussian noise. Our major contribution consists in deriving the optimal noise level required to obtain the maximum capacity for a given threshold decision level. Based on the work derived in this paper one can find the maximum capacity of the BAC/SE in the context of SR. We use the Helgert and Pinsker capacity bound approximations [3],[4],[8] to study the capacity behaviour for a distributed cooperative wireless sensor underwater network.

The paper is organized as follows. In Section II we provide the background for the mutual information and capacity of the (2, 3) DMC. The effects of the SR and the physical model of the (2, 3) DMC communication channel are analyzed in Section III. In Section IV we derive the optimal noise level and the optimal threshold level in order to obtain the maximum capacity. The conclusions and future work are given in Section V

2. The (2,3) DMC Mutual Information and Capacity

The (2,3) DMC $\{M: X \rightarrow Y\}$ is characterized by a binary input random variable $X = \{1, -1\}$, a ternary output random variable $Y = \{1, 0, -1\}$ and a (2×3) channel transition probability matrix M defined by $M = \begin{pmatrix} p_{11} & p_{21} & p_{31} \\ p_{12} & p_{22} & p_{32} \end{pmatrix}$. The entries of this matrix are the p_{ij} which represent the conditional probability between each output and input values as represented in Fig.1.

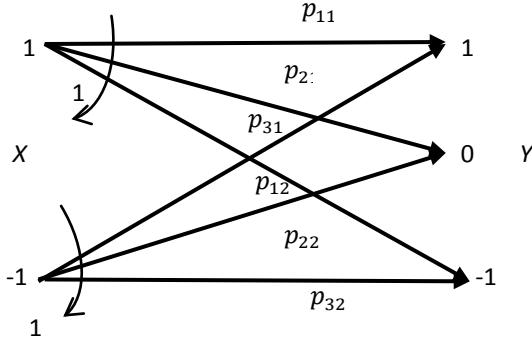


Figure 1. (2, 3) DMC

Associated with Fig.1 we have the following conditional probabilities between binary input and ternary output:

$$\begin{aligned} p_{11} &= P(Y = 1|X = 1), \quad p_{21} = P(Y = 0|X = 1), \\ p_{31} &= P(Y = -1|X = 1), \quad p_{12} = P(Y = 1|X = -1), \\ p_{22} &= P(Y = 0|X = -1), \quad p_{32} = P(Y = -1|X = -1). \end{aligned}$$

Since M is a stochastic matrix we have, $p_{31} = 1 - p_{11} - p_{21}$ and $p_{32} = 1 - p_{12} - p_{22}$.

Defining $P(X = i) = p(x_i)$, $i = 1, 2$, and $P(Y = j) = p(y_j)$, $j = 1, 2, 3$, then the mutual information between the binary input X and the ternary output Y is

$$I(X, Y) = H(Y) - H(Y|X) \quad (1)$$

where $H(Y)$ the entropy function of a Y probability distribution defined as

$$H(Y) = -\sum_j p(y_j) \log_2 p(y_j) \quad (2)$$

and $H(Y/X)$, the conditional entropy defined by the following relation

$$H(Y/X) = \sum_{i,j} p(x_i, y_j) \log_2 \frac{p(x_i)}{p(x_i, y_j)} \quad (3)$$

Using (2) and (3), we can rewrite the mutual information as follows

$$I(X, Y) = \sum_{i,j} p(x_i, y_j) \log_2 \frac{p(x_i, y_j)}{p(x_i) \cdot p(y_j)} \quad (4)$$

Since $p(x_i) \cdot p(y_j|x_i) = p(x_i, y_j)$ and $p(y_j) = \sum_{k=1}^2 p(y_j|x_k) p(x_k)$ we finally have

$$I(X, Y) =$$

$$\sum_{i=1}^2 \sum_{j=1}^3 p(x_i) p(y_j|x_i) \log_2 \frac{p(y_j|x_i)}{\sum_{k=1}^2 p(x_k) p(y_j|x_k)} \quad (5)$$

In order to calculate the capacity of the (2, 3) DMC channel, we want to maximize the mutual information $I(X, Y)$ [6], [10]. Based on (5), we see that $I(X, Y)$ is a function of $p(x_1)$ and $p(x_2)$ since the $p(y_j|x_i) = p_{ji}$ are fixed: $I(X, Y) = f(p(x_1), p(x_2))$. Therefore, in order to calculate the capacity of this channel, we need to maximize $I(X, Y)$ over all distributions of the form $(p(x_1), p(x_2))$. Since $p(x_1) + p(x_2) = 1$, we set $x = p(x_1)$, $p(x_2) = 1 - x$, giving

$$C(X, Y) = \max_x I(X, Y) \quad (6)$$

and finally replacing (5) in (6) we have

$$\begin{aligned} C(X, Y) &= \\ \max_x \sum_{i=1}^2 \sum_{j=1}^3 p(x_i) p(y_j|x_i) \log_2 &\frac{p(y_j|x_i)}{\sum_{k=1}^2 p(x_k) p(y_j|x_k)} \end{aligned} \quad (7)$$

While the general capacity solution for (6) is not known, in [7], a closed form solution is given for the capacity of the binary asymmetric erasure channel with symmetric erasure (BAC/SE). The BAC/SE is a particular case of the (2, 3) DMC with the following constraints: $p_{21} = p_{22}$ and $p_{11} > p_{12}$ (without loss of generality) as mentioned in [7].

In [7], using the Kuhn-Tucker condition, the following solution is found for the BAC/SE channel capacity:

$$C(X, Y) =$$

$$\sum_{j=1}^3 p(y_j|x) \log_2 \frac{p(y_j|x)}{x(p(y_j|x_1) - p(y_j|x_2)) + p(y_j|x_2)} \quad (8)$$

where

$$x = \frac{kp_{32} - p_{12}}{p_{11} - p_{12} - k(p_{31} - p_{32})} \quad (9)$$

where

$$k = \left(\frac{p_{11}p_{11}p_{31}p_{31}}{p_{12}p_{12}p_{32}p_{32}} \right)^{\frac{1}{p_{11}-p_{12}}} \quad (10)$$

Using the closed form solution (8), we will examine the stochastic resonance that arise in the (2,3) BAC/SE channels.

3. Stochastic Resonance

Following the work of [13] we will examine what is the effect of AGN on the capacity of the (2,3) BAC/SE (as shown above) derived in [7]. The input to the threshold communication channel is the signal x that takes the binary values -1 and 1 with the probabilities $p(x_1)$ and $p(x_2)$. The physical model is represented in Fig.2 where in the presence of the AGN n , the received signal y is given by

$$y = x + n \quad (11)$$

The physical communication channel presented in Fig.2 contains a threshold decision block function $t(\theta)$, where θ represents the threshold value. Based on the threshold level θ and the noise level, the received signal y is converted into a discrete random variable y taking on the values of 1, 0, or -1.

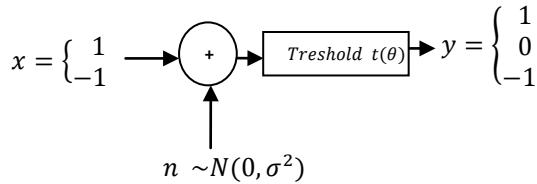


Figure 2: Physical model of binary-input ternary-output channel with zero mean Gaussian noise

For the ternary output, the threshold function is defined as follows:

$$t(\theta) = \begin{cases} -1 & x + n < -\theta \\ 0 & -\theta \leq x + n \leq \theta \\ 1 & x + n > \theta \end{cases} \quad (12)$$

By using the threshold level we will determine the analytic expression for each of the conditional probabilities $p_{11}, p_{21}, p_{31}, p_{12}, p_{22}, p_{32}$ between the binary input and the ternary output. We assumed the noise n to have a Gaussian distribution with mean $\mu = 0$ and variance $\sigma > 0$ with the following cumulative density distribution

$$F_n(\theta) = P_r(n \leq \theta) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\theta} e^{-\frac{t^2}{2\sigma^2}} dt \quad (13)$$

Therefore we have

$$\begin{aligned} p_{11} &= P(1|1) = P(Y = 1|X = 1) = \\ \Pr(x + n > \theta | x = 1) &= 1 - \Pr(n < -1 + \theta) = \\ 1 - F_N(-1 + \theta) & \end{aligned} \quad (14)$$

Using a similar approach we have

$$\begin{cases} p_{21} = P(0|1) = F_N(-1 + \theta) - F_N(-1 - \theta) \\ p_{31} = P(-1|1) = F_N(-1 - \theta) \\ p_{12} = P(1|-1) = 1 - F_N(1 + \theta) \\ p_{22} = P(0|-1) = F_N(1 + \theta) - F_N(1 - \theta) \\ p_{32} = P(-1|-1) = F_N(1 - \theta) \end{cases} \quad (15)$$

Since $F(-x) = 1 - F(x)$, and, according with (14) and (15) the following relation stands

$$p_{11} = p_{32}, p_{21} = p_{22}, p_{31} = p_{12} \quad (16)$$

When noise is modeled as a standard Gaussian distribution, and since $1 - \theta > -1 - \theta$, we obtain that $F_N(1 - \theta) > F_N(-1 - \theta)$ and $F_N(1 + \theta) - F_N(1 - \theta) = F_N(\theta - 1) - F_N(-1 - \theta)$, respectively. These are exactly the values which correspond respectively to $p_{11} > p_{12}$ and $p_{21} = p_{22}$ which characterize the capacity of the BAC/SE. At this point, we make the fundamental observation that the (2,3) physical communication channel presented in Fig.2 is a BAC/SE channel in the presence of AGN. This motivates our work to study the capacity SR behavior of this channel which was not presented in [11]. The effect of other types of noise on this communication channel is left for future work.

Using (13), (14) and (15), we can re-write the (2, 3) DMC matrix as:

$$M = \begin{pmatrix} p_{11} & p_{21} & p_{31} \\ p_{12} & p_{22} & p_{32} \end{pmatrix} = \begin{pmatrix} 1 - F_N(-1 + \theta) & \\ & 1 - F_N(1 + \theta) \end{pmatrix} \begin{pmatrix} F_N(-1 + \theta) - F_N(-1 - \theta) & F_N(-1 - \theta) \\ F_N(1 + \theta) - F_N(1 - \theta) & F_N(1 - \theta) \end{pmatrix} \quad (17)$$

By using (16) we have $k = 1$ in (9) and $x = 1/2$ in (10).

The capacity of the BAC/SE channel could then be rewritten below in (18) and it is plotted in Fig.3 and Fig.4:

$$\begin{aligned} C(X, Y) &= \\ F_N(1 - \theta) \log_2 \left(\frac{F_N(1 - \theta)}{0.5(F_N(1 - \theta) - F_N(-1 - \theta)) + F_N(-1 - \theta)} \right) &+ \\ F_N(-1 - \theta) \log_2 \left(\frac{F_N(-1 - \theta)}{0.5(F_N(-1 - \theta) - F_N(1 - \theta)) + F_N(1 - \theta)} \right) & \end{aligned} \quad (18)$$

We observe the phenomenon of SR on capacity over a specific range of θ and noise power σ as it is plotted in Fig.3.

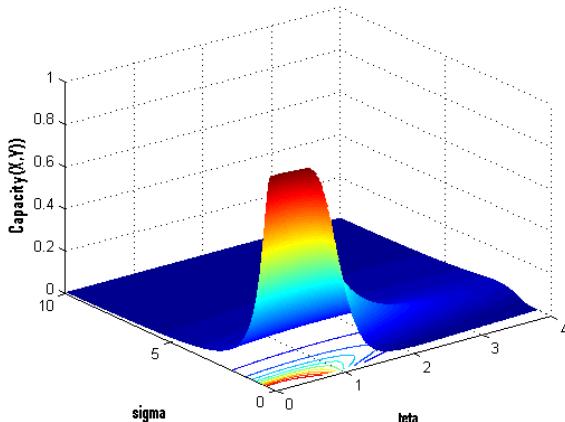


Figure 3: Capacity of BAC/SE as a function of θ and σ

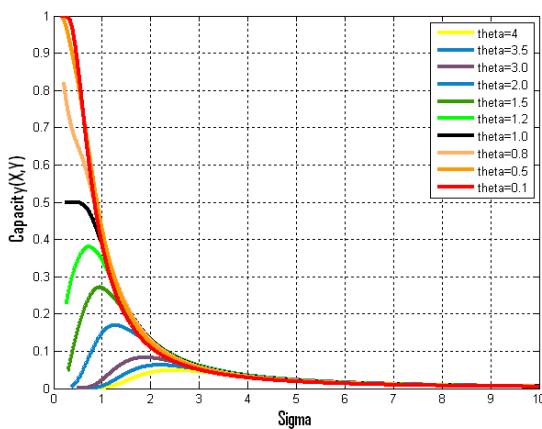


Figure 4. Capacity of BAC/SE for $\theta = \{0.5, 0.6, 1, 1.5, 2, 2.5, 3, 3.5, 4\}$

To be more specific it is important to note that the SR appears only for $\theta > 1$ as it is plotted in Fig.4. For the noise power $\sigma \in [5,10]$ and the threshold level $\theta > 3$ in Fig. 3, the capacity is vanishing to zero. Similar results were reported before in [1], [2], and [9] for the (2,2) DMC. While these results are in concordance with the intuition that increasing the noise power on the channel will decrease the capacity, the opposite, when the noise power decreases the capacity is also decreasing will be detailed in the full paper.

4. Stochastic Resonance and Capacity Bounds for (2,n) DMC

Due to the non-existence of a closed form expression for a (2,n) capacity channel in general and, the non-linear expression of the (2,3) capacity channel in particular, we will use the bounds (upper and lower) developed in [4] for the particular case of the (2,3) DMC in order to examine the phenomenon of stochastic resonance. In [4],[8] it has

been proven that any (2, n) discrete memoryless channel with 2 inputs and n outputs with the transition matrix $M = \begin{pmatrix} p_{1,1} & \dots & p_{n-1,1} & p_{n,1} \\ p_{1,2} & \dots & p_{n-1,2} & p_{n,2} \end{pmatrix}$, where $p_{n,1} = 1 - p_{1,1} - \dots - p_{n-1,1}$ and $p_{n,2} = 1 - p_{1,2} - \dots - p_{n-1,2}$, can be approximated by : a lower bound called the Pinsker bound, that we denote by L, and a upper bound, the Helgert bound, that we denote U. The Pinsker and Helgert bounds are defined as follow:

$$L \leq C \leq U \text{ where}$$

$$L = \frac{1}{8 \ln(2)} \left[\left(\sum_{i=1}^{n-1} |p_{i,1} - p_{i,2}| \right) + |p_{n,1} - p_{n,2}| \right]^2$$

$$U = \max \left(\sum_{i=1}^{n-1} p_{i,1} ; \sum_{i=1}^{n-1} p_{i,2} \right) - \left(\sum_{i=1}^{n-1} \min(p_{i,1} ; p_{i,2}) \right)$$

For a (2,3) DMC the above expressions are reduced to

$$\begin{aligned} & \frac{1}{8 \ln(2)} [|p_{11} - p_{12}| + |p_{21} - p_{22}| + |p_{31} - p_{32}|]^2 \\ & \geq C \geq \\ & \max(p_{11} + p_{21}; p_{12} + p_{22}) - \min(p_{11}; p_{12}) - \\ & \min(p_{21}; p_{22}) \end{aligned} \quad (24)$$

We have shown previously that using the threshold model given in Fig.2 and modeling noise as a standard Gaussian distribution we have the following equality:

$$p_{11} = p_{32}, p_{21} = p_{22}, p_{31} = p_{12}, p_{21} = p_{22}, p_{11} \geq p_{12}, \text{ and } p_{32} \geq p_{31}$$

Using these relations and (24) we have:

$$\frac{1}{2 \ln(2)} (p_{11} - p_{12})^2 \leq C \leq p_{11} - p_{12} \quad (25)$$

Using (14) and (15) we have

$$\frac{1}{2 \ln(2)} (F_N(1 - \theta) - F_N(-1 - \theta))^2 \leq C \leq F_N(1 - \theta) - F_N(-1 - \theta) \quad (26)$$

Finally we have

$$L = \frac{1}{2 \ln(2)} (F_N(1 - \theta) - F_N(-1 - \theta))^2 = f(\theta, \sigma)$$

$$U = F_N(1 - \theta) - F_N(-1 - \theta) = g(\theta, \sigma)$$

In Fig.5a, Fig.5b, and Fig. 6, we noted that, just as mentioned in [7], the bounds capture the increasing/decreasing behaviour of the capacity and exhibit stochastic resonance for the threshold level $\theta > 1$.

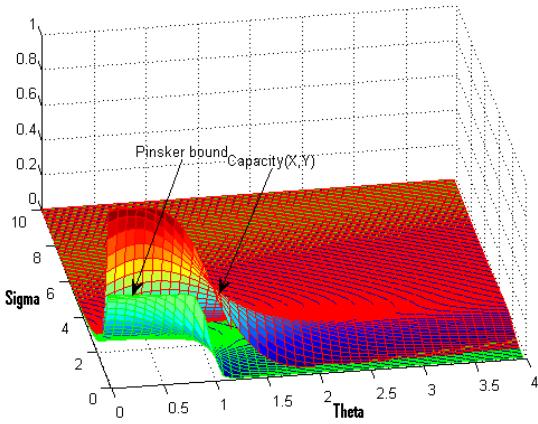


Figure 5.a (2, 3) DMC and Pinsker capacity Bound

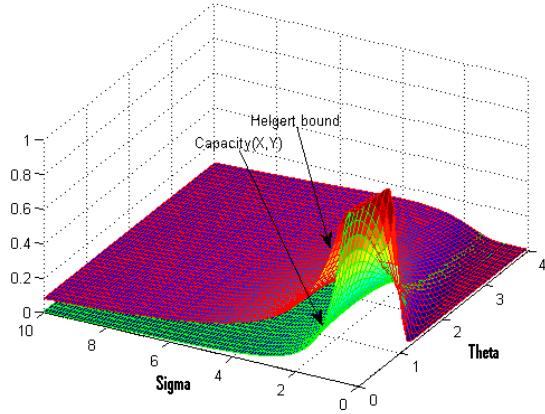


Figure 5.b (2, 3) DMC and Pinsker capacity Bound

Following similar work done in [3], we will use the bounds to find the optimum noise given a threshold that will maximize the capacity. The above lower and upper bounds are given by:

$$L = \frac{1}{2 \ln(2)} (F_N(1 - \theta) - F_N(-1 - \theta))^2 \text{ and}$$

$$U = F_N(1 - \theta) - F_N(-1 - \theta).$$

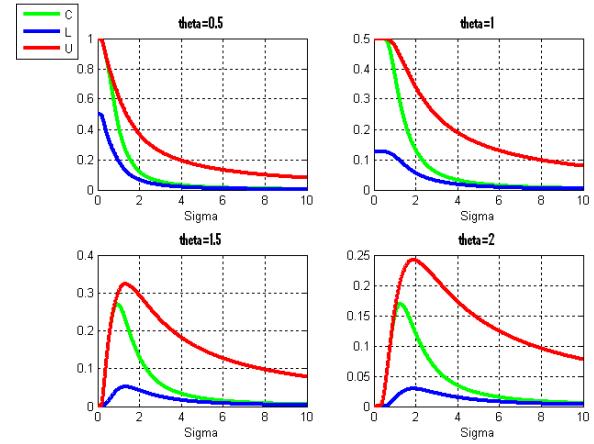


Figure 6. Capacity(C), Pinsker (L) and Helgert (U) bounds for different values of θ

5. A Optimal Values for Threshold and Noise Level

We are looking for the optimal values of correlated noise and threshold level. As in [3], our goal is to find the critical points of the capacity bounds. This will require us to take the partial derivatives $\frac{d}{d\sigma} L(\theta, \sigma)$ and $\frac{d}{d\sigma} U(\theta, \sigma)$, and find the solution to the equations $\frac{d}{d\sigma} L(\theta, \sigma) = 0$ and $\frac{d}{d\sigma} U(\theta, \sigma) = 0$ respectively.

The partial derivative of the bounds with respect to σ will have the same critical points because:

$$\begin{aligned} \frac{d}{d\sigma} L(\theta, \sigma) &= \frac{1}{\ln(2)} (F_N(1 - \theta) - F_N(-1 - \theta))' \cdot (F_N(1 - \theta) - F_N(-1 - \theta)) \\ &= \frac{1}{\ln(2)} \frac{d}{d\sigma} U(\theta, \sigma) \cdot (F_N(1 - \theta) - F_N(-1 - \theta)) \end{aligned}$$

Thus, solving $\frac{d}{d\sigma} L(\theta, \sigma) = 0$ is equivalent to solving $\frac{d}{d\sigma} U(\theta, \sigma) = 0$ which implies that they both have the same critical points. Therefore we will only focus on $\frac{d}{d\sigma} U(\theta, \sigma)$. We have

$$\begin{aligned} \frac{d}{d\sigma} U(\theta, \sigma) &= \frac{d}{d\sigma} (F_N(1 - \theta) - F_N(-1 - \theta)) = \\ &= \frac{d}{d\sigma} \frac{1}{\sigma \sqrt{(2\pi)}} \int_{-1-\theta}^{1-\theta} e^{-\frac{t^2}{2\sigma^2}} dt = \end{aligned}$$

$$\frac{-1}{\sigma^2 \sqrt{(2\pi)}} [(1-\theta)e^{\frac{-(1-\theta)^2}{2\sigma^2}} - (-1-\theta)e^{\frac{-(1+\theta)^2}{2\sigma^2}}]$$

Solving for σ , $\frac{d}{d\sigma} U(\theta, \sigma) = 0$, we will obtain

$$\ln\left(\frac{1+\theta}{1-\theta}\right) = \frac{-4\theta}{2\sigma^2}$$

For $\theta > 1$ and $\sigma > 0$, we finally have:

$$\sigma = \sqrt{\frac{2\theta}{\ln\left(\frac{1+\theta}{1-\theta}\right)}} \quad (27)$$

The above relation gives us the optimum power noise σ as a function of θ . In Fig.8, we plotted (in blue) $\sigma = f(\theta)$ and (in red) $f^{-1}(\theta)$. The plots depicted in Fig.7 will provide two opportunities to improve the capacity channel; either by adding or reducing noise depending on the threshold or by simply adjusting the threshold depending on the noise level. It is important to mention that from the plot of Fig.7, we see that as θ increases, $\sigma \rightarrow \theta$. That is because $\lim_{\theta \rightarrow \infty} \sigma - \theta = 0$, which means that $\theta = \sigma$ is an oblique asymptote to $\sigma = f(\theta)$ and $\theta = f'(\sigma)$ as plotted in Fig.7:

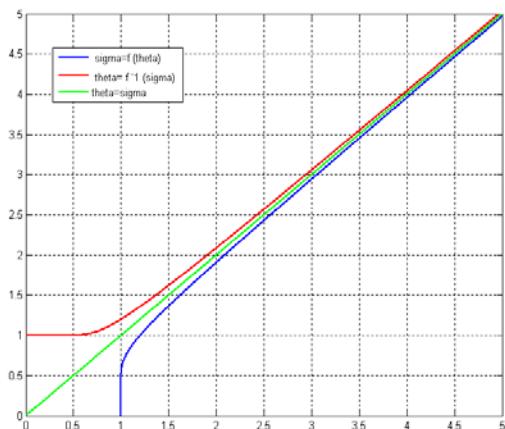


Figure 7: Optimal noise power and threshold for stochastic resonance, with a scalable factor

5. Conclusion

In this paper we have studied the threshold based stochastic resonance behaviour of binary-input ternary-output discrete memoryless channel. We proved that in the presence of the AWGN the (2, 3) DMC become a BAC/SE channel for which the capacity channel behaviour were analyzed in the context of the stochastic resonance phenomenon. For the (2, n) DMC capacity

behaviour, we used the lower and upper capacity bounds, we also have illustrated the stochastic resonance. Based on these bounds, we have derived analytically the optimum noise power level depending on the threshold level that will maximize their capacity. This paper clearly indicates how one can improve the capacity of binary threshold communication channel by changing the noise power or the threshold level. In the context of the underwater communications, where noncoherent communication techniques are affected both by noise and threshold level we derive the optimal noise power and threshold for stochastic resonance, with a scalable factor. This research is a step forward toward the design of efficient sensors that can operate in very noisy environment. In future work we will focus on the phenomenon of stochastic resonance in non Gaussian channel.

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